● a) $z \neq -i$, on a :

$$z'+i = \frac{1-z}{1-iz} + i = \frac{1-z+i(1-iz)}{1-iz} = \frac{1-z+i+z}{1-iz}$$

$$= \frac{1+i}{(-i)(z+i)} = \frac{-1+i}{z+i}$$

D'où $z'+i = \dfrac{-1+i}{z+i}$

b) On a pour $z \neq -i$: $z'+i = \dfrac{-1+i}{z+i}$

Donc $(z'+i)(z+i) = -1+i$

$\Leftrightarrow |(z'+i)(z+i)| = |-1+i|$

$\Leftrightarrow |(z'+i)|.|(z+i)| = |-1+i|$

$\Leftrightarrow |(z'-(-i)|.|z-(-i)| = \sqrt{2}$

$\Leftrightarrow |(z'-z_B|.|z-z_B| = \sqrt{2}$

$\Leftrightarrow BM' \times BM = \sqrt{2}$

c) M décrit le cercle de centre B et de rayon 1

Signifie que : BM = 1 et comme $BM' \times BM = \sqrt{2}$

Alors on aura $BM' = \sqrt{2}$ d'où le point M' décrit le cercle de centre B et de rayon $\sqrt{2}$

Exercice n° 11

● • On sait que : $\cos 3x = \mathrm{Rel}(e^{i3x})$

Or on a : $e^{i3x} = (e^{ix})^3 = (\cos x + i \sin x)^3$

$e^{i3x} = \cos^3 x + 3i\cos^2 x.\sin x - 3\cos x.\sin x - i\sin^3 x$

$= \cos^3 x - 3\cos x.\sin x + i(3\cos^2 x.\sin x - \sin^3 x)$

Donc : $\boxed{\cos 3x = \cos^3 x - 3\cos x.\ \sin x}$

• On sait que : $\sin 5x = \mathrm{Im}(e^{i5x})$

Or on a : $e^{i5x} = (e^{ix})^5 = (\cos x + i\sin x)^5$

$e^{i5x} = \cos^5 x + 5i\cos^4 x.\sin x - 10\cos^3 x.\sin^2 x - 10i\cos^2 x.\sin^3 x + 5\cos x.\sin^4 x + i\sin^5 x$

Donc :

$$\boxed{\sin 5x = 5\cos^4 x.\sin x - 10\cos^2 x.\sin^3 x + \sin^5 x}$$

● $\cos^3 x = \left(\dfrac{e^{ix}+e^{-ix}}{2}\right)^3$

$$= \frac{e^{i3x}+3e^{2ix}e^{-ix}+3e^{ix}e^{-2ix}+e^{-3ix}}{8}$$

$$= \frac{1}{4}.\frac{e^{i3x}+e^{-3ix}}{2} + \frac{3}{4}.\frac{e^{ix}+e^{-ix}}{2}$$

Par suite :

$$\boxed{\cos^3 x = \frac{1}{4}\cos 3x + \frac{3}{4}\cos x}$$

• $\cos^2 x.\sin x = (1-\sin^2 x).\sin x = \sin x - \sin^3 x$

or on a :

$\sin^3 x = \left(\dfrac{e^{ix}-e^{-ix}}{2i}\right)^3$

$$= \frac{e^{i3x}-3e^{2ix}e^{-ix}+3e^{ix}e^{-2ix}-e^{-3ix}}{-8i}$$

$$= -\frac{1}{4}.\frac{e^{i3x}-e^{-3ix}}{2i} + \frac{3}{4}.\frac{e^{ix}-e^{-ix}}{2i}$$

$$= -\frac{1}{4}\sin 3x + \frac{3}{4}\sin x$$

D'où : $\cos^2 x.\sin x = \sin x - (-\frac{1}{4}\sin 3x + \frac{3}{4}\sin x)$

$$\boxed{\cos^2 x.\sin x = \frac{1}{4}(\sin x + \sin 3x)}$$

• $\sin^2 x.\cos^3 x = (1-\cos^2 x).\cos^3 x = \cos^3 x - \cos^5 x$

on a déjà: $\cos^3 x = \frac{1}{4}\cos 3x + \frac{3}{4}\cos x$

Linéarisons : $\cos^5 x$

$\cos^5 x = \left(\dfrac{e^{ix}+e^{-ix}}{2}\right)^5 =$

$$\frac{e^{i5x}+5e^{4ix}e^{-ix}+10e^{3ix}e^{-2ix}+10e^{2ix}e^{-3ix}+5e^{ix}e^{-4ix}+e^{-i5x}}{32}$$

$$= \frac{1}{16}.\frac{e^{i5x}+e^{-5ix}}{2} + \frac{5}{16}.\frac{e^{3ix}+e^{-3ix}}{2} + \frac{10}{16}.\frac{e^{ix}+e^{-ix}}{2}$$

$$= \frac{1}{16}\cos 5x + \frac{5}{16}\cos 3x + \frac{5}{8}\cos x$$

$\sin^2 x.\cos^3 x = \frac{1}{4}\cos 3x + \frac{3}{4}\cos x - (\frac{1}{16}\cos 5x + \frac{5}{16}\cos 3x + \frac{5}{8}\cos x)$

$$= -\frac{1}{16}\cos 5x - \frac{1}{16}\cos 3x + \frac{1}{8}\cos x$$

● $\alpha \in \mathbb{R}$ et $\beta \in \mathbb{R}$

$Z = \dfrac{e^{i\alpha}+e^{i\beta}}{e^{i\alpha}-e^{i\beta}}$ est définit si $e^{i\alpha}-e^{i\beta} \neq 0$ c'est à dire

$e^{i\alpha} \neq e^{i\beta}$ donc pour $\alpha \neq \beta + 2k\pi$, $k \in \mathbb{Z}$

On a : $\dfrac{e^{i\alpha}+e^{i\beta}}{e^{i\alpha}-e^{i\beta}} = \dfrac{e^{i(\frac{\alpha+\beta}{2})}(e^{i(\frac{\alpha-\beta}{2})}+e^{-i(\frac{\alpha-\beta}{2})})}{e^{i(\frac{\alpha+\beta}{2})}(e^{i(\frac{\alpha-\beta}{2})}-e^{i(\frac{\beta-\alpha}{2})})}$

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